Electrical Impedance Tomography, inverse problems, material characterization & structural health monitoring

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UEF // University of Eastern Finland
SIR,

WE'RE SURROUNDED.
SIR, WE'RE SURROUNDED

EXCELLENT
NOW WE CAN FIRE IN ANY DIRECTION
SIR, WE'RE SURROUNDED
EXCELLENT
NOW WE CAN FIRE IN ANY DIRECTION
Inverse problems
Linnanmäki
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Electrical Impedance Tomography (EIT)

Ill-posedness of the EIT inverse problem

EIT-imaging of concrete

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Electrical Impedance Tomography (EIT)

- In EIT electric currents $I$ are applied to electrodes on the surface of the object and the resulting potentials $V$ are measured using the same electrodes.

- The conductivity distribution $\sigma = \sigma(x)$ is reconstructed based on the potential measurements.
The forward model in EIT

\[
\nabla \cdot (\sigma \nabla u) = 0, \quad x \in \Omega \\
u + z_\ell \sigma \frac{\partial u}{\partial \nu} = U^{(\ell)}, \quad x \in e_\ell, \ l = 1, 2, \ldots, L \\
\int_{e_\ell} \sigma \frac{\partial u}{\partial \nu} dS = I^{(\ell)}, \quad \ell = 1, 2, \ldots, L \\
\sigma \frac{\partial u}{\partial \nu} = 0, \quad x \in \partial \Omega \setminus \bigcup_{\ell=1}^L e_\ell
\]
Finite element approximation of the EIT forward model

FE-approximation of the complete electrode model \( \Rightarrow \)

\[ V = U(\sigma) \]

where \( \sigma \in \mathbb{R}^N \) is a finite dimensional approximation of the conductivity.
Solution of the forward problem
Solution of the forward problem
Solution of the forward problem
Solution of the forward problem
Solution of the forward problem
Solution of the forward problem
Solution of the forward problem
Two different targets & electrode potentials
Inverse problem of EIT
Two different targets & electrode potentials
Two different targets & electrode potentials
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Inverse problem of EIT
Modeling errors in EIT

Example 1: Modeling error due to unknown contact impedances $z$ (true $z = 1$, assumed $z = 0.01$).

Left: True conductivity distribution. Middle: EIT reconstruction based on correct model ($z=1$). Right: EIT reconstruction based on incorrect model ($z=0.01$).
Modeling errors in EIT

Example 2: Modeling error due to inaccuracy in the injected current $I$ (error level in $I$: 0.5%).

Left: True conductivity distribution. Middle: EIT reconstruction based on correct model. Right: EIT reconstruction based on incorrect model.
Modeling errors in EIT

- Example 3: Modeling error due to unknown boundary shape.

Left: Photograph of a target. Middle: EIT reconstruction based on correct geometry. Right: EIT reconstruction based on circular model geometry.

- Nissinen et al 2010
Inverse problem of EIT
Solution of the inverse problem of EIT

- Least-squares fitting?

\[ \sigma_{LS} = \arg \min_\sigma \{ \| V_{obs} - U(\sigma) \|^2 \} \]
Solution of the inverse problem of EIT

- Reconstructing the conductivity $\sigma$ based on noisy observations $V_{\text{obs}}$ is an *ill-posed inverse problem*.

- The solution of the inverse problem is typically written in the form

$$\sigma_{\text{MAP}} = \arg \min_{\sigma > 0} \{ \| L_n(V_{\text{obs}} - U(\sigma)) \|^2 + A(\sigma) \}$$

- The functional $A(\sigma)$ models the *prior information* on the conductivity distribution $\sigma$.

- Non-linear, constrained optimization problem
Iteration step 1

Left: estimated conductivity distribution. Right: Measured vs. computed potentials.
Iteration step 2

Left: estimated conductivity distribution. Right: Measured vs. computed potentials.
Iteration step 3

Left: estimated conductivity distribution. Right: Measured vs. computed potentials.
Iteration step 4

Left: estimated conductivity distribution. Right: Measured vs. computed potentials.
Iteration step 5

Left: estimated conductivity distribution. Right: Measured vs. computed potentials.
Final estimate

Figure: Left: Photo of the true target; Right: estimated conductivity distribution.
The resolution of EIT is usually not very high...
However, if feasible prior information on the resistivity is available, the resolution can be improved...
“Blobology”

- However, if feasible prior information on the resistivity is available, the resolution can be improved...
Concrete

- The most extensively used construction material in the world
- About 7.5 cubic kilometers of concrete made each year
- In the United States, more than 55,000 miles of highways paved with concrete
- $35-billion industry. In the United States, 2 million workers
- Evaluation, repair and restoration: 35% of the total volume work in building industry
U.S. roads, bridges are decaying despite stimulus influx

Gary Stoller, USA TODAY  12:13 a.m. EDT July 29, 2013

NEW FAIRFIELD, CONN. — As car after car is jolted by cracked asphalt on a less-than-1-mile stretch of road connecting Route 39 to the New York border, it becomes clear why state transportation officials grade the pavement of this winding western Connecticut road as being in poor condition.

Edges of the two-lane road — where a sign says Col. Henry Ludington passed by in 1777 to repel "British raiders" — are worn and recessed, allowing rainwater to pool.

Connecticut has the nation's second-highest percentage of major roads — 48%, or 1,268 miles — with pavement in "poor" condition, and 25 other states have 20% or more in such condition, according to an exclusive analysis of the Federal Highway Administration's (FHWA) most recent data by transportation research group TRIP and USA TODAY.

Indeed, just 38% of the pavement on roads stretching miles across the USA is in "good" condition, according to the analysis, while about one in 10 of the nation's bridges are "structurally deficient."
U.S. BRIDGES FALLING DOWN

In May, a bridge classified as functionally obsolete — the Interstate 5 bridge over the Skagit River in Washington — collapsed after a truck struck a girder. No one died or was seriously injured. The National Transportation Safety Board is investigating the cause. (Photo: Mike Siegel, AP)
Concrete, need for evaluation/testing/monitoring

- On-site testing & evaluation
  - Crack detection
  - Prediction of rebar corrosion risk, etc...

- Material characterization in lab scale
  - Evaluation of transport properties – esp. the ability of concrete to impede the ingress of water
EIT imaging of concrete
EIT imaging of 3D moisture flow in concrete
ECT imaging of 3D moisture flow in concrete
ECT imaging of 3D moisture flow in concrete

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<th>time - 36 h</th>
<th>time - 48 h</th>
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<td>time - 72 h</td>
<td>time - 120 h</td>
<td>time - 240 h</td>
</tr>
</tbody>
</table>
ECT imaging of 3D moisture flow in concrete
Results with X-ray CT...
Imaging of cracks
EIT-based sensing skin for damage detection

- Electrically conductive material (e.g. copper tape, CNT film, copper/silver paint) is applied on the surface of concrete.
- The cracking of concrete breaks also the sensing skin.
- Detecting of cracks in the surface material with EIT.
EIT-based sensing skin for damage detection

- We choose the painted sensing skin (Easy to apply & applicable to a large scale).
- 2D EIT imaging problem
Case 1: Sensing skin on plexi-glass

- Sensing skin painted on plexi-glass
- 16 electrodes for EIT
- Synthetic cracks made by scratching the paint
Case 1
Case 1
Case 1
Case 1
Case 1
Case 1
Case 1
Case 1
Case 1
No blobology!
Case 1: difference vs absolute reconstructions
How?

- We fit homogeneous conductivity distribution $\sigma_{\text{ref}}$ to reference EIT data $V_{\text{ref}}$
- Denote the discrepancy between $V_{\text{ref}}$ and the modeled data by $\epsilon$

$$\epsilon = V_{\text{ref}} - U(\sigma_{\text{ref}})$$

This error is mostly due to inhomogeneity of the sensing skin.

- An approximative modeling error correction; observation model

$$V = U(\sigma) + \epsilon + n$$
How?

MAP estimate

\[ \sigma_{\text{MAP}} = \arg \min_{0<\sigma<\sigma_{\text{ref}}} \left\{ \frac{1}{2} \| L_n(V - U(\sigma) - \epsilon) \|^2 + A(\sigma) \right\} \]

where \( A(\sigma) \) is a potential function related to a total variation prior

\[ A(\sigma) = \alpha \int_\Omega \| \nabla \sigma \| dr \]

\( A(\sigma) \) promotes sparsity of \( \nabla \sigma \).
Case 2: Notched concrete beam in 4-point bending
Case 2: Notched concrete beam in 4-point bending
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Case 2: Notched concrete beam in 4-point bending
Case 2: Photo vs. EIT reconstruction
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Case 2: Photo vs. EIT reconstruction
Case 2: Photo vs. EIT reconstruction
Case 2: Photo vs. EIT reconstruction
Case 2: reconstructions, denser FE mesh
Temperature sensing experiment

- Sensing skin was exposed to temperature changes by contact with a heat source.
- Temperature of the heat source could be controlled within 2°C, when in contact with the temperature sensor.
- Reconstructed conductivities were converted to temperature maps based on an experimentally determined T vs. σ curve.
Local temperature change 77°C
Local temperature changes 37°C and 77°C
A trans-European Consortium to Assess, Improve and Regulate Promising Sub-Surface Geo-Energy Technologies via the Reliable Quantification of Environmental Footprint and Risks
H2020 project, Science for clean energy
H2020 project, Science for clean energy
Thank you!