Exercises, VK

1. Let
$$\pi(x) \propto \exp\left\{-10(x_1^2 - x_2)^2 - (x_2 - \frac{1}{4})^4\right\}, \quad x \in \mathbb{R}^2. \tag{1}$$
Solve the ratio $q(x', x)/q(x, x')$ which is needed when solving the Metropolis-Hastings acceptance ratio
$$\alpha(x, x') = \min\left\{1, \frac{\pi(x')q(x', x)}{\pi(x)q(x, x')}\right\}$$
when elements x_1' and x_2' of the proposal x' are drawn independently such that
$$x_i' = x_i + \epsilon_i, \epsilon_i \sim \mathcal{N}(0, \gamma^2),\ i = 1, 2,$$
where

2. Implement the assignment 1 (Matlab,...).

3. (Assignment 1 continued) Solve the ratio $q(x', x)/q(x, x')$ when elements x_1' and x_2' of the proposal x' are drawn such that
$$x_i' = x_i + \delta_i(x) + \epsilon_i, \epsilon_i \sim \mathcal{N}(0, \gamma^2),\ i = 1, 2,$$
where $\delta_i : \mathbb{R}^2 \mapsto \mathbb{R}$ is a deterministic mapping.

4. Implement the assignment 3 when we choose
$$\delta_i(x) = h\frac{\partial \ln \pi(x)}{\partial x_i} \tag{2}$$
where step parameter $h = \text{constant}$. Investigate the effect of the parameters (h, ϵ) on the behaviour of the algorithm.